ABSTRACT

Global-scale wave climate models, such as WAVEWATCH III, are widely used in oceanography to hindcast the sea state that occurred in a particular geographic area at a particular time. These models are applied in rogue-wave science for characterizing the sea states associated with observations of rogue waves (e.g., the well known “Draupner” [1] or “Andrea” [2] waves). While spectral models are generally successful in providing realistic representations of the sea state and are able to handle a large number of physical factors, they are also based on a very coarse grained representation of the wave field and therefore unsuitable for a detailed resolution of the wave field and refined wave-height statistics.

On the other hand, local wave models based on first-principle fluid dynamics equations (such as the Higher Order Spectral Method) are able to represent wave fields in detail, but in general they are hard to interface with the full complexity of real-world sea conditions. This paper displays our efforts in coupling these two types of models in order to enhance our understanding of past extreme events and provide scope for rogue wave risk evaluation. In particular, high resolution numerical simulations of a wave field similar to the “Andrea” wave one are performed, allowing for accurate analysis of the event.

INTRODUCTION

Over the course of the last decade, the topic of “Rogue-Waves” has been subjected to intense study, with much ground being covered theoretically, experimentally, and numerically, providing an understanding of the physics behind the emergence of these freakishly large waves. Reviews have been presented, for example, by Kharif et al. [3], Dysthe et al. [4], and Adcock et al. [5]. One particular area of interest in rogue wave science is that of sea states contributing to the generation of such waves, whose time evolution can be followed and analysed to provide various wave height statistics and sea state parameters. Various techniques are employed in this pursuit, each with their own perks, and of course drawbacks.

Global scale climate models based on the conservation of
wave action are frequently employed in oceanography to perform large scale hindcasts and forecasts of the wind-wave directional spectrum describing a particular sea state of interest. Examples include the WAM (Komen et al. [6]) model and WAVEWATCH III (Tolman et al. [7]). Such wave spectral models have proven to yield satisfactory simulations for large spatial domains over long periods of time, providing key sea state parameters such as significant wave height, directional spreading, wind velocity, mean wave direction, etc. However, such models are limited to describing the sea state as two dimensional, coarse grained wave fields, which are evolved over large time steps, and so they are unable to produce the instantaneous surface position at a given time.

Conversely, local scale wave models bridge the gap. The evolution of the free surface elevation of an incompressible fluid may be described by the Euler equations, which while complicated can be numerically solved locally via, for example, pseudo spectral methods in three dimensions, and conformal mapping methods in two. Of the former, the higher order spectral method (HOSM), is of note, particularly for studying rogue wave dynamics ([8], [9]). Such local scale models are not subject to the resolution constraints of wave spectral models, and so are capable of simulating the time evolution of the desired sea state at extremely fine resolution and small integral time step, albeit at the cost of smaller physical domain and shorter time scale.

Bitner-Gregersen et al. [10] have performed HOSM type simulations initialised with a hindcasted directional spectrum pertaining to the Andrea event, provided by the European Centre for Medium Range Weather Forecast (ECMWF), and in doing so display potential for coupling for local and global scale models. Their work suggests that the Andrea sea state is capable of triggering Benjamin-Feir type instabilities. Our investigation endeavours to perform such a coupling, in order to provide a thorough examination of the desired sea state. In particular, the HOSM and WAVEWATCH III models are implemented. Our HOSM simulations are initialised with a directional spectrum characterised by the ECMWF Andrea wave hindcast. However, due to the availability of only a deep water code for the time being, no definite conclusions can be drawn relative to the Andrea event. Indeed, the Andrea wave is characterized by a period of 13.2s in about 75m water depth. Consequently, the wavelength ($\approx 262m$) is more than twice the water depth and it is a wave in intermediate water depth. Note that by considering it as a deep water wave, the modulational instability is exaggerated. Still our conclusions show that the modulational instability does not play a major role. So the main conclusion for the Andrea wave, to be checked by using a finite depth code, is that the modulational instability played a minor role in the formation of the rogue wave.

![Table 1](https://i.imgur.com/5Q5Q5Q5.png)

**TABLE 1.** Important characteristics of the Andrea Wave

**ANDREA WAVE**

The Andrea Wave was recorded at the Ekofisk platform complex in the North Sea on the 9th of November, 2007, during the Andrea Storm passing through the area at the time. Magnusson et al. [2] have given a detailed examination of the event itself; here we include the pertinent facts. On the 8th, a low pressure field entering the northern region of the North sea, and strong westerly wind field of 50-55 knots enters the area. As a result, a high wave field was built up around the north area of the Ekofisk platform, with significant wave heights of 10-11m being recording in the evening. At 18:00 UTC, the pressure field moves from southern Norway to south eastern Sweden early on the 9th. The wind field itself passes from the north of the Ekofisk field to being located East of the field at 06:00 UTC on the 9th. Significant wave heights of 11-12m were recorded at this time, and wind speeds around the time of the Andrea event were approximately 22ms$^{-1}$.

The Andrea wave itself was recorded just after 00:00 UTC on the 9th, by down looking Optech lasers on site at the Ekofisk platform, at which water depth is 75m. See table 1 for important characteristics of the event. Of particular note are the ratios of maximum crest height and maximum wave height to significant wave height, $CH_{max}/H_s$ and $H_{max}/H_s$ respectively. A wave is considered to be a rogue wave if either $CH_{max}/H_s > 1.25$ or $H_{max}/H_s > 2$, both of which are satisfied by the characteristics of the Andrea wave, and so it is called a double rogue wave. The wave steepness is less than the breaking threshold, $\pi/7$ ([11], [12]), so wave breaking will not be a prominent feature. Finally, the figures for peak period and depth indicate a peak wavelength $\lambda_p \approx 262m$, and so we can surmise that the event occurred between intermediate and deep water depth. The HOSM model, however, was constructed assuming deep water, as explained above.

Hindcast data pertaining to the Andrea wave event was obtained from the ECMWF data archive to be used as an initial condition for the HOSM model. Contained within this data is
the directional wave spectrum, \( E(\theta, t) \), describing the Andrea sea state, which is stored at 6 hour intervals over the period 00:00 UTC 8th 10th November, 2007. The spectrum (Fig 1) at 00:00 UTC on the 9th November is extracted for analysis, it being the closest time point to the Andrea event.

**MATHEMATICAL FRAMEWORK**

**Higher Order Spectral Method**

The flow of an inviscid, incompressible and irrotational body of water is conveniently described by the velocity potential \( \phi(x,y,z,t) \), which satisfies the Laplace equation everywhere within the domain of the fluid. In addition, on the free surface \( \eta(x,y,t) \), the velocity potential also satisfies a kinematic and dynamic boundary condition pertaining to conservation of energy and momentum, and so collectively we have the Euler equations describing the dynamics of the fluid flow in deep water:

\[
\nabla^2 \phi = 0, \quad \text{for} \quad -\infty < z < \eta(x,y,t) \tag{1}
\]

\[
\phi_t + g \eta + \frac{1}{2} (\nabla \phi)^2 = 0, \quad \text{at} \quad z = \eta(x,y,t) \tag{2}
\]

\[
\eta_t + \nabla_h \phi \cdot \nabla_h \eta = \phi_z, \quad \text{at} \quad z = \eta(x,y,t) \tag{3}
\]

\( \nabla_h \) is the gradient operator in the x-y plane. Defining the velocity potential on the free surface as

\[
\psi(x,y,t) = \phi(x,y,z = \eta,t), \tag{4}
\]

the free surface boundary conditions (Eqns. (2) - (3)) may be rewritten as

\[
\psi_t + g \eta + \frac{1}{2} (\nabla_h \psi)^2 - \frac{1}{2} \eta^2 \left( 1 + (\nabla_h \eta)^2 \right) = 0, \tag{5}
\]

\[
\eta_t + \nabla_h \psi \cdot \nabla_h \eta - W \left( 1 + (\nabla_h \eta)^2 \right) = 0, \tag{6}
\]

where

\[
W = \frac{\partial \phi}{\partial z} |_{z = \eta(x,y,t)}, \tag{7}
\]

denotes the vertical velocity on the free surface.

The HOSM was developed independently by Dommermuth & Yue [13] and by West et al. [14] in 1987 to numerically solve the Euler equations. Each version differs slightly in their method for determining \( W \); West et al.’s version has superior consistency between numerical and analytical results, however, and as such their version is the one used in the present work. For a full discussion on the method, see Tanaka [15]; herein follows a brief description. Using an initial condition for \( \eta \) and \( \phi \), the HOSM directly solves the Laplace equation for \( \phi \) at each time step by assuming a periodic series expansion in the wave steepness \( \varepsilon \) as solution. A series solution for \( W \) is then evaluated, which allows the time evolution of the \( \eta \) and \( \psi \) to be followed via Eqns. (5)-(6). The nonlinear terms present in Eqns. (5)-(6) are smoothly ramped up via the Dommermuth ramping function [16] over ramp time \( T_r \approx O(5)T_p \), as in Xiao et al. [8].

A third order expansion including four wave interactions (as per Janssen, [17]) is used, and it has been shown in [15] to be equivalent physically to Zakharov’s Hamiltonian formalism (see also [24]). Nonlinear interactions between free and bound wave modes are an intrinsic feature to the Euler equations, making the HOSM quite suitable for studying nonlinear rogue wave generation mechanisms, (e.g. Benjamin-Feir type instabilities). Kurtosis and skewness of surface elevation distribution are important statistical features to examine, particularly kurtosis, which is understood to be highly relevant to the study of rogue wave generation. Kurtosis values above Gaussian result in distribution functions with higher tails corresponding to freak events. Mori & Janssen [21] have shown that a relationship exists between kurtosis and the square of the Benjamin-Feir index, which is a measure of a wave’s susceptibility to modulational (Benjamin-Feir) instability, and this relationship is linear for narrowband spectra. Mori et al. and Xiao et al. have put forward that directionally spread spectra, which, as shown in Fig. 1, is a feature of the Andrea field, exhibit damped kurtosis values (and indeed damped BFI).

With regards to limitations, \( \eta \) and \( \psi \) are represented by a Fourier series within the model, and as such wave overturning cannot be reproduced by the HOSM. Therefore, care must be
taken when applying it to a particular wave field; if the wave field is large enough to feature breaking prominently, results from the HOSM may not be accurate. This does not apply to the Andrea wave field, however, whose steepness is below the breaking criterion. The model does not include wind effects on wave development, which presents limitations in choice of time/length scales.

**WAVEWATCH III**

WAVEWATCH III (Chawla et al. [18]; Tolman [7]) is a third generation wave model developed at NOAA/NCEP. The model solves for the random phase spectral action density balance equation (8) for wave number-direction spectra and is updated constantly with the latest innovative new physics, numerical and other improvements developments.

\[ N(k, \theta) \equiv \frac{|E(k, \theta)|^2}{\omega}, \quad (8) \]

where \( E(k, \theta) \) is the wavenumber-direction spectrum, and \( \omega \) is the intrinsic radian frequency. Wave propagation is then described by the following balance equation:

\[ \frac{\partial N}{\partial t} + \mathbf{k} \cdot \mathbf{x} N + \frac{\partial}{\partial k} k N + \frac{\partial}{\partial \theta} \theta N = S \quad (9) \]

where

\[ \mathbf{x} = c_g + U, \]

\[ \mathbf{k} = -\frac{\partial \omega}{\partial c} \frac{d}{dc} \mathbf{U} - k \cdot \frac{\partial \mathbf{U}}{\partial \alpha}, \]

\[ \theta = -\frac{\partial \omega}{\partial c} \frac{d}{dc} \mathbf{U} + k \cdot \frac{\partial \mathbf{U}}{\partial \beta}. \]

Here, \( c_g \) is the group wave velocity, \( d \) is mean water depth, \( U \) is current velocity, and \( \alpha \& \beta \) are coordinates in the direction of \( \theta \& \alpha \), respectively. The \( S \) term represents the net effect of the various sources and sinks to the spectrum, which has three main contributions:

\[ S_{in} \ - \ Wind \ input \ (linear \ and \ exponential), \]

\[ S_{nl} \ - \ Nonlinear \ Interactions \ (4-wave, \ resonant), \]

\[ S_{ds} \ - \ Dissipation \ (whitecapping), \]

and so

\[ S = S_{in} + S_{nl} + S_{ds} + ... \]

\[ \frac{\partial \mathbf{N}}{\partial t} + \mathbf{k} \cdot \mathbf{x} N + \frac{\partial}{\partial k} k N + \frac{\partial}{\partial \theta} \theta N = S \]

Finally, it must be noted that the spectrum output by the model is the frequency-directional power spectrum, \(|E(\omega, \theta)|^2\).

**NUMERICAL SIMULATIONS**

**Higher Order Spectral Method**

From the directional spectrum \( E(\omega, \theta) \), one may construct initial conditions for \( \eta \) and \( \psi \) for use with the HOSM. First, the spectrum must be interpolated over the higher resolution grid used by the HOSM, and random phase introduced:

\[ E(\omega, \theta) \rightarrow E(\omega, \theta) \exp(i\beta), \quad (10) \]

where \( \beta \) is normally distributed over \([0, 2\pi]\). The spectrum is then converted to the wavenumber domain via the deepwater dispersion relation, and normalised with respect to the conversion. This yields the wavenumber spectrum for \( \eta \):

\[ \hat{\eta}(k_x, k_y) = \frac{1}{2} \frac{g^{1/2}}{|k|^{3/2}} (E(k_x, k_y) + c.c.), \quad (11) \]

where \( c.c. \) denotes the complex conjugate. \( \hat{\psi} \) is easily obtained via linear wave theory, and, finally, performing an inverse FFT on both \( \hat{\eta} \) and \( \hat{\psi} \) yields the initial condition for \( \eta \) and \( \psi \) (Fig 2). As such, the HOSM is initialised as a linear wave field, with non-linear terms smoothly ramped up as described earlier.

To ensure accurate simulations of the Andrea wave field by the HOSM, appropriate domains and runtimes must be selected with respect to the physics being considered. As such,
timescales are chosen in accordance with the Benjamin-Feir timescale ([20], [8]), $T/\tau_p \sim O(\varepsilon^{-2})$. Similarly, physical domain size is chosen so that $L_{\text{max}}k_p \sim O(\varepsilon^{-2})$. $k_p$ and $T_p$ are the peak wavenumber and peak wave period, respectively, and our convention for wave steepness is $\varepsilon = k_p H_\text{s}/2$. Dysthe et al. have pointed out that for wave age $(C_p/U_{10})$ above a certain threshold ($\approx 0.42$), wind effects are not significant on this timescale. The considered sea state has a deep water phase speed $C_p = 20.6$ ms$^{-1}$, and so for wind speeds recorded at the time of the event, wind effects need not be considered over the BF timescale.

A large number of Fourier nodes is used in order to produce refined wave statistics and a high resolution simulation of wave field. Also, as the directional spectrum needs to be interpolated to the HOSM’s resolution, many Fourier nodes are required to ensure important parameters of the wave field are not warped. As such, 1024 x 1024 modes are used. Time integration was handled via a Dormand-Prince-Shampine Runge-Kutta method, with time step set to $\Delta t = 0.02$ s to minimise energy leakage.

The free surface of the Andrea wave field on the 9th of November at 00:00 UTC, as simulated by HOSM, can be seen in fig 3, and refined statistical analysis of the wave field can be seen in fig 4. As can be seen from the statistical analysis, the sea state resembles a steady quasi Gaussian state. Kurtosis and skewness measured from the HOSM output are both consistent with the variability seen with physical wave fields. This damped kurtosis implies low BFI, and so the generation of rogue waves as a result of modulational instability is unlikely for this particular sea state. Although weakly non Gaussian values are seen (maximum excess kurtosis is approximately 0.1), freakish deviations from the surface height distribution may still occur, and are seen in plot of maximum crest heights in Fig 4. Finally, the positively weakly non Gaussian skewness, indicating a bias in the surface elevation distribution above the significant wave height.

FIGURE 4. Statistical analysis of wave field simulated by HOSM over BF timescale. The initial ‘jump’ is a result of the linear initial condition coupled with the ramping up of nonlinear terms.

IMPLEMENTATION OF WAVEWATCH III

The configuration of WAVEWATCH III (version 4.18) grid model covers a region of the North Atlantic $80^\circ$N, $18^\circ$W, $30^\circ$E, at spatial resolution of $0.25^\circ$. The bathymetry grid data comes from the GEODAS NOAA’s National Geophysical Data Centre (NGDC), with a resolution of 1 minute of degree in latitude and longitude. It was linearly interpolated spatially to the $0.25^\circ$ model grid. The wave spectrum is provided for 36 directional bands, and 30 frequencies from the minimum frequency of 0.0350 Hz up to 0.5552 Hz. JONSWAP spectrum at every grid point was set as initial condition with the following parameters: Phillips’ parameters 0.18; peak frequency 0.2 Hz; overshoot factor 3.0; left peak width 0.07; right peak width 0.09; wave direction $0^\circ$; fetch 30 000 metres. The reanalysis used for this study come from the Climate Forecast System Reanalysis (CFSR) from NOAA [22]. The CFSR is a third generation reanalysis product. The CFSR global atmosphere resolution is about 38 km with 64 levels. The simulations for the Andrea wave began at 1st September 2007 lasting two months and 15 days up to 15th November 2007. CFSR wind fields have been linearly interpolated in space to $0.25^\circ$ to match with the spatial resolution of the WAVEWATCH III grid. The wave hindcast temporal resolution was set to 1 hourly wind input/output time step.

Hindcast output can be seen in Figs. 6 (time series) and 5 (spectral evolution). Although the Andrea event was the primary concern for this investigation, studying the time series and spectral evolution may yield other potentially interesting times to look at with the HOSM. For instance, times with relatively lower directional spreading, or interesting spectral features (for example, the presence of two spectral peaks at the beginning of October), which while not influential to the Andrea event, may provide fuel for other studies.
CONCLUSIVE REMARKS

In the present work we have displayed the capability of coupling local scale and global scale wave models in studying oceanic sea states, with particular application to rogue wave science. In particular, we used the wave spectral model WAVEWATCH III, and the HOSM free surface model. In order to display the power of such coupling of wave models, we have studied a deep water sea state characterised by the Andrea wave, which was recorded at the Ekofisk platform in the North sea on the 9th of November, just passed 00:00 UTC. Although both models are computationally intensive, significant advances are being made in computational power, and so the coupling is viable from a performance standpoint.

The advantage of the coupling is clear; global scale models can examine over large periods of time and large domains the evolution of the sea state with reasonable accuracy, providing information on key aspects of the wave field, such as mean wave direction, significant wave height, etc, and accounts for long term effects such as wind-forcing. Periods/locations of particular interest may then be examined in finer detail using local scale models, providing a high resolution reconstruction of the wave field, from which refined analysis of the sea state can be performed. Although ECMWF hindcast data was used to construct initial conditions for the HOSM (a result of the emphasis being placed on the Andrea wave), wave spectral models may provide interesting directional spectra to be used with HOSM type simula-
tions instead. Overall, this allows for rigorous investigation of the evolution of the sea state in question, and can be applied to both forecasts and hindcasts.

As emphasized in the introduction, our simulations cannot be quantitatively related to the Andrea event since we used a deep water HOSM code. However, our HOSM simulations resemble that of a steady quasi Gaussian sea state, with statistical analysis showing weakly non Gaussian values of skewness and kurtosis. These results imply that the Benjamin-Feir instability may not have been a strong contributing factor to the development of rogue waves in such a sea state. Similar results and conclusions were presented by Xiao et al., who performed HOSM type simulations on a broadband JONSWAP spectrum in deep water. We plan to extend our code to finite depth and to show results in finite depth at the conference.

ACKNOWLEDGEMENTS

This work is supported by the European Research Council (ERC) under the research project ERC-2011-AdG 290562-MULTIWAVE and Science Foundation Ireland under grant number SFI/12/ERC/E2227. The authors would also like to acknowledge Claudio Viotti for his contributions to the development of the HOSM code, and the Irish Centre for High-End Computing for access to the Fionn supercomputer system.

REFERENCES


[22] Saha Suranjana, S., Moorthi Pan, H.L., Wu X., Wang J., Nadiga S., Tripp P., Kistler R.,
